Numeric Response Questions

Straight Line

Q.1 A square of area 25sq. unit is formed by taking two sides as $3x+4y=kt$ and $3x+4y=kz$, then find value of $|k_1-k_3|$.

Q.2 If the lines $3y+4x=1, y=x+5$ and $5y+bx=3$ are concurrent, then find the value of b.

Q.3 A straight line passing through $P(3,1)$ meet the coordinate axes at A and B. It is given that distance of this straight line from the origin ' O ' is maximum. Then find area of triangle OAB.

Q.4 A variable line $x/a+y/b=1$ moves in such a way that harmonic mean of a and b is 8 . Then find the least area of triangle made by the line with the coordinate axes.

Q.5 Let ABC be a triangle. Let A be the point $(1,2)$, $y=x$ is the perpendicular bisector of AB and $x-2y+1=0$ is the angle bisector of $\angle C$. If equation of BC is given by $ax+by-5=0$, then find the value of $a+b$,

Q.6 Find the number of straight lines parallel to $3x+6y+7=0$ & have intercept of length 10 between the coordinate axes.

Q.7 Find the distance of origin from line $(1+\sqrt{3})y+(1-\sqrt{3})x=10$ along the line $y=\sqrt{3}x+k$.

Q.8 If the distance between the purallel lines $y=2x+4$ and $6x=3y+5$ is $\frac{k\sqrt{5}}{15}$ then find k.

Q.9 If the lines $x=a+m, y=-2$ and $y=mx$ are concurrent, then find the least value of a^2,

Q.10 Reflection of a point $(t-1,2t+2)$ in a line is $(2t+1,t)$, then find the slope of line.

Q.11 The acute angle bisector between the lines $2x-y+4=0$ and $x-2y-1=0$ is $y-x=k$, then find value of ' k ',

Q.12 Find the sum of the abscissas of the points lying on the line $x-y=3$, which lies at a unit distance from $4x-3y=12$

Q.13 If $(\sin\ \theta, \cos\ \theta), \theta \in [0,2\pi]$ and (1,4) lie on the same side or on the line $\sqrt{3}x - y + 1 = 0$, then find the maximum value of $\sin\ \theta$.

Q.14 If all lines given by the equation $(3\sin\ \theta + 5\cos\ \theta)x + (7\sin\ \theta - 3\cos\ \theta)y + 11(\sin\ \theta - \cos\ \theta) = 0$ pass through a fixed point P for all $\theta \in R$, then find the half distance of P from $Q(7, -10)$.

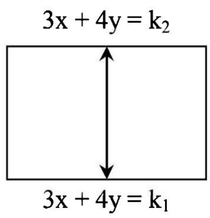
Q.15 Let A = (3,4) and B is a variable point on the lines $| \times | = 6$, If AB $\leq$ 4, then find the number of positions of B with integral coordinates.

ANSWER KEY

1. 25.00	**2.** 6.00	**3.** 16.66	**4.** 32.00	**5.** 2.00	**6.** 2.00	**7.** 5.00
8. 17.00	**9.** 8.00	**10.** 1.00	**11.** 1.00	**12.** 6.00	**13.** 0.00	**14.** 5.00
15. 5.00						

Hints & Solutions

1.



Side $d = \left|\frac{k_1 - k_2}{\sqrt{9+16}}\right|$

Area = 25

$d^2 = 25$

$\left(\frac{k_1 - k_2}{5}\right)^2 = 25$

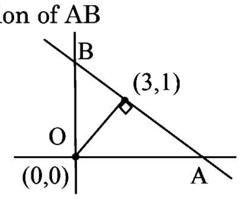
$\Rightarrow \ |k_1 - k_2| = 25$

2. $\begin{vmatrix} 3 & 4 & -1 \\ 1 & -1 & -5 \\ 5 & b & -3 \end{vmatrix} = 0$

$3(3 + 5b) - 4(-3 + 25) - 1(b + 5) = 0$

$\Rightarrow \ b = 6$

3. equation of AB



$y - 1 = -3\ (x - 3)$

$y - 1 = -3(x - 3)$

$3x + y = 10$

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$\Delta = \frac{1}{2} \times \frac{10}{3} \times 10 = \frac{100}{6}$ sq.unit

4. a, 8, b are in H.P.

$\Rightarrow \ \frac{1}{a} + \frac{1}{b} = \frac{1}{4}$

$\Rightarrow \ b = \frac{4a}{a-4}$

$\Rightarrow$ area, $A = \frac{4a^2}{2(a-4)}$

A is minimum at a = 8. Hence, minimum value of A is 32 sq. units.

5. The point B is (2, 1)

Image of A (1, 2) in the line $x - 2y + 1 = 0$ is given

by $\frac{x-1}{1} = \frac{y-2}{-2} = \frac{4}{5}$

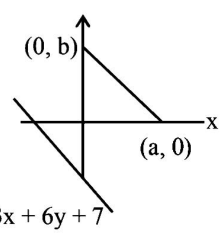
$\therefore$ coordinate of the point are $\left(\frac{9}{5}, \frac{2}{5}\right)$

Since this point lies on BC.

$\therefore$ equation of BC is $3x - y - 5 = 0$

$\therefore a + b = 2$

6.



$\because$ Let line parallel to given line has slope $= -b/a$

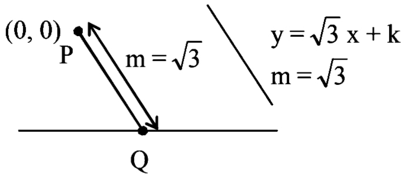
$\therefore$ given $-\frac{b}{a} = -\frac{1}{2} \Rightarrow a = 2b$ ...(1)

($\therefore$ a & b are of same sign)

Now $a^2 + b^2 = 100 \Rightarrow 5b^2 = 100$

$\Rightarrow \ b = \pm\sqrt{20} \ \Rightarrow a = \pm\ 2\sqrt{20}$

7. $(1+\sqrt{3})\,y + (1-\sqrt{3})\,x = 10$



Q$(0 + r\cos 60^\circ,\ 0 + r\sin 60^\circ)$

$$(1+\sqrt{3})\left(\frac{r\sqrt{3}}{2}\right) + (1-\sqrt{3})\left(\frac{r}{2}\right) = 10$$

$$\frac{r}{2}\{\sqrt{3} + 3 + 1 - \sqrt{3}\} = 10$$

$2r = 10 \Rightarrow r = 5$

8. $y = 2x + 4$

$\Rightarrow 2x - y + 4 = 0$ .... (1)

and $6x = 3y + 5$

$\Rightarrow 6x - 3y - 5 = 0$

$\Rightarrow 2x - y - \frac{5}{3} = 0$ ...... (2)

$$d = \left|\frac{c_1 - c_2}{\sqrt{a^2+b^2}}\right| = \left|\frac{4+5/3}{\sqrt{4+1}}\right|$$

$$= \frac{17}{3\sqrt{5}} = \frac{17\sqrt{5}}{15}$$

9. Eliminating x and y from the three equations, we get

$-2 = m(a + m) \Rightarrow m^2 + am + 2 = 0.$

Since $m \in R$,

discriminant ≥ 0

$\Rightarrow a^2 - 8 \geq 0$

$\Rightarrow a^2 \geq 8$

10. $\therefore$ Slope of line joining given points is

$$= \frac{2t+2-t}{t-1-(2t+1)} = \frac{t+2}{-2-t} = -1$$

$\therefore$ Slope of perpendicular line = 1

11. $2x - y + 4 = 0$ and $-x + 2y + 1 = 0$

Now $a_1a_2 + b_1b_2 = 2(-1) + (-1)2 = -4 < 0$

hence '+" sign gives the acute angle bisector.

$$\frac{2x-y+4}{\sqrt{5}} = +\frac{-x+2y+1}{\sqrt{5}}$$

$\Rightarrow 3x - 3y = -3$

$\Rightarrow x - y = -1$

12. Let the point is $(a, a - 3)$

$$\Rightarrow \left|\frac{4a-3(a-3)-12}{5}\right| = 1$$

$\Rightarrow |a - 3| = 5$

$\Rightarrow a = 8, -2$

$\therefore$ sum = 6

13. $\because \sqrt{3}(1) - (4) + 1 < 0$

$\therefore \sqrt{3}\sin\theta - \cos\theta + 1 > 0$

14. $(3x + 7y + 11)\sin\theta + (5x - 3y - 11)\cos\theta = 0$

$\Rightarrow$ point is their intersection point $(L_1 + \lambda L_2 = 0)$

solving, $3x + 7y + 11 = 0$ and $5x - 3y - 11 = 0$

Point 'P' is $(1, -2)$ & Q$(7, -10)$

$$\therefore \frac{PQ}{2} = \frac{\sqrt{6^2+8^2}}{2} = 5$$

15. $B = (\pm 6, y)$. So, $AB \leq 4$

$\Rightarrow (3 \mp 6)^2 + (y - 4)^2 \leq 16$

$\therefore 9 + (y - 4)^2 \leq 16$,

($\because 81 + (y - 4)^2 \leq 16$ is absurd)

$\Rightarrow y^2 - 8y + 9 \leq 0$

$\Rightarrow 4 - \sqrt{7} \leq y \leq 4 + \sqrt{7}$

But y is an integer.

$\Rightarrow y = 2, 3, 4, 5, 6$